Assignment 3 – First Order Logic

1. Express the following plain English sentences in First Order Logic by using the predicate definitions below.

Predicate	Meaning
$Happy(x)$	"x is happy"
$Vegetarian(x)$	"x is a vegetarian"
$Likes(x, y)$	"x likes y"

Predicate	Meaning
$Female(x)$	"x is a female"
$Fisherman(x)$	"x is a fisherman"
$Eats(x, y)$	"x eats y"

The domain of x and y is the set of all people.

1.1 Mickey and Minnie are both not happy.

Correct Answer: $\neg Happy(Mickey) \wedge \neg Happy(Minnie)$
Incorrect Answers:

- $\neg\Big[Happy(Mickey) \wedge Happy(Minnie)\Big]$ means '*One of them maybe happy*'.
- $Happy(Mickey) \wedge Happy(Minnie)$ means '*Both of them are happy*'.
- $\neg\Big[Happy(Mickey, Minnie)\Big]$ is a wrong use of *Happy* predicate.

1.2 Mickey and Minnie are not both happy.

Correct Answer:
$\neg\Big[Happy(Mickey) \wedge Happy(Minnie)\Big] \equiv \neg Happy(Mickey) \vee \neg Happy(Minnie)$
Incorrect Answers:

- $Happy(Mickey) \vee Happy(Minnie)$ means '*Both of them maybe happy*'.
- $\Big(\neg Happy(Mickey) \wedge Happy(Minnie)\Big) \vee \Big(Happy(Mickey) \wedge \neg Happy(Minnie)\Big)$ does not include the case when they are both not happy.

1.3 Minnie is happy if and only if Mickey is happy.

Correct Answer:
$Happy(Mickey) \leftrightarrow Happy(Minnie)$
$\equiv \Big(Happy(Mickey) \rightarrow Happy(Minnie)\Big) \wedge \Big(Happy(Minnie) \rightarrow Happy(Mickey)\Big)$
Incorrect Answers:

- $Happy(Minnie) \rightarrow Happy(Mickey)$ means '*Mickey is happy if Minnie is happy, but not the other way around*'.

1.4 Everybody likes all vegetarians.

Correct Answer:

$\forall x \forall y \Big(\mathit{Vegetarian}(y) \rightarrow \mathit{Likes}(x, y) \Big)$

Incorrect Answers:

- $\forall x \forall y \Big(\mathit{Likes}(x, y) \rightarrow \mathit{Vegetarian}(y) \Big)$ means '*All people whom everybody likes are vegetarian*'. Any other kind of people are not liked.
- $\forall x \forall y \Big(\mathit{Likes}(x, y) \land \mathit{Vegetarian}(y) \Big)$ means '*All people like everybody, and all people are vegetarian*'.
- $\forall x \forall y \Big(\mathit{Likes}(x, \mathit{Vegetarian}(y)) \Big)$ is a wrong FOL expression. *Likes* predicate needs two objects, but *Vegetarian* predicate returns a truth value not an object.

1.5 All people who do not eat meat likes all vegetarians.

Correct Answer:

$\forall x \forall y \Big((\mathit{Vegetarian}(y) \land \neg \mathit{Eats}(x, \mathit{Meat})) \rightarrow \mathit{Likes}(x, y) \Big)$

Incorrect Answers:

- $\forall x \forall y \Big(\neg \mathit{Eats}(x, \mathit{Meat}) \land \big(\mathit{Likes}(x, y) \rightarrow \mathit{Vegetarian}(y)\big) \Big)$ means '*All people do not eat meat, and all people whom everybody likes are vegetarian*'. They do not like any other kind of people.
- $\forall x \forall y \Big(\neg \mathit{Eats}(x, \mathit{Meat}) \land \mathit{Likes}(x, y) \land \mathit{Vegetarian}(y) \Big)$ means '*All people do not eat meat, like everybody, and are vegetarian*'.

1.6 Some vegetarians are fishermen.

Correct Answer:

$\exists x \Big(\mathit{Vegetarian}(x) \land \mathit{Fisherman}(x) \Big)$

Incorrect Answers:

- $\exists x \Big(\mathit{Fisherman}(x) \rightarrow \mathit{Vegetarian}(x) \Big)$ means '*There exists a person who is not a fisherman or is a vegetarian*'.

1.7 Some fishermen do not like fish.

Correct Answer:

$\exists x \Big(\mathit{Fisherman}(x) \land \neg \mathit{Likes}(x, \mathit{Fish}) \Big)$

1.8 Nobody likes a woman who is vegetarian.

Correct Answer:

$\neg \Big[\exists x \exists y \Big(\mathit{Vegetarian}(y) \land \mathit{Likes}(x, y) \Big) \Big]$

1.9 Not all fishermen who are vegetarian are happy.

Correct Answer:
$\neg\left[\forall x\Big((Fisherman(x) \land Vegetarian(x)) \rightarrow Happy(x)\Big)\right]$

1.10 Mickey does not like any vegetarian.

Correct Answer:
$\forall x\Big(Vegetarian(x) \rightarrow \neg Likes(Mickey, x)\Big)$

2. Let $L(x, y)$ be a predicate "x loves y". The domain of x and y is the set of all people. Translate to following First Order Logic sentences into plain English.

2.1 $\forall x \exists y\Big(L(x, y) \rightarrow L(y, x)\Big)$

Everybody has somebody that they loves, and that person also loves them back.

2.2 $\exists x \exists y \exists z\Big(L(x, y) \land L(x, z) \land \neg(y = z) \land \forall w\Big(L(x, w) \rightarrow ((w = y) \lor (w = z))\Big)\Big)$

There exists somebody who loves exact two persons.

2.3 $\exists y \forall x\Big(L(x, y) \land (L(y, Kitty) \lor L(y, Browny))\Big)$

There exists somebody whom everybody loves and that person loves Kitty or Browny.