

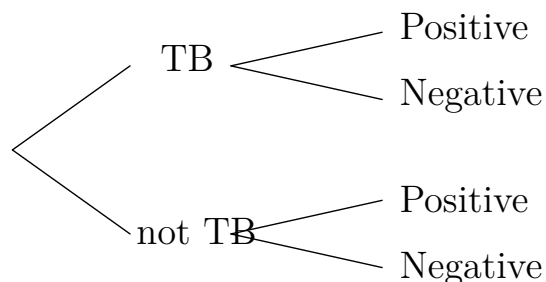
ITS336 Lecture 10.

Tree Diagrams and Bayes' Theorem

10.1. Tree Diagrams

In solving many probability problems, it is helpful to represent the various events and their associated probabilities by a tree diagram. To explain this useful notion, suppose that we wish to compute the probability of an event that results from performing a sequence of experiments. The various outcomes of each experiment are represented as branches emanating from a point.

Example 10.1 Suppose that the reliability of a skin test for active pulmonary tuberculosis (TB) is specified as follows: Of people with TB, 98% have a positive reaction and 2% have a negative reaction; of people free of TB, 99% have a negative reaction and 1% have a positive reaction. From a large population of which 2 per 10,000 persons have TB, a person is selected at random and given a skin test, which turns out to be positive. What is the probability that the person has active pulmonary tuberculosis?

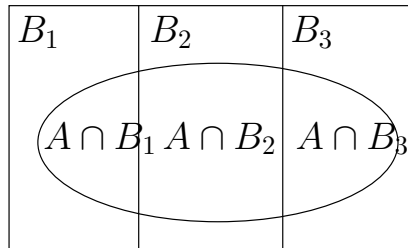


3. What is the probability that a student selected at random received an A in the statistics course?

4. What is the conditional probability that a student previously took a finite mathematics course, given that he or she received an A in the statistics course?

10.2. Bayes' Theorem

If B_1, B_2, \dots, B_k is a partition of a sample space U of outcomes, and if A is viewed as the set of successful outcomes of the experiment, then we visualize the situation as the figure below.



This means that

$$p(A) = p(B_1)p(A|B_1) + p(B_2)p(A|B_2) + \dots + p(B_k)p(A|B_k)$$

When substitute this into the conditional probability, we get

$$p(B_i|A) = \frac{p(B_i)p(A|B_i)}{p(B_1)p(A|B_1) + p(B_2)p(A|B_2) + \dots + p(B_k)p(A|B_k)}$$

This formula for $p(B_i|A)$ is called **Bayes' Theorem**.

$$\begin{array}{l}
 p(B_1) \swarrow B_1 \frac{p(A|B_1)}{p(A)} A \quad p(B_1 \cap A) \\
 p(B_2) \quad B_2 \frac{p(A|B_2)}{p(A)} A \quad p(B_2 \cap A) \\
 p(B_3) \searrow B_3 \frac{p(A|B_3)}{p(A)} A \quad p(B_3 \cap A)
 \end{array}$$

Exercise 10.2 Solve example 10.1 using Bayes' Theorem.

Exercise 10.3 A population in which 3% are users of a drug is screened by a test that is 80% accurate in the sense that the probability of a false-positive (tested positive, given non-drug user) is 0.20 and the probability of a false-negative (tested negative, given drug user) is also 0.20. If a person test positive, calculate the probability that he or she is, in fact, not a user of the drug?

Exercise 10.4 It is estimated that 10% of Olympic athletes use steroids. The test currently being used to detect steroids is said to be 93% effective in correctly detecting steroids in users. It yields false positives in only 2% of the tests. A country's best weightlifter tests positive. What is the probability that he actually takes steroids?

Exercise 10.5 There are two cookie jars on the shelf in the kitchen. The red one has 10 chocolate-chip cookies and 15 gingersnaps. The blue jar has 20 chocolate-chip cookies and 10 gingersnaps. James goes down in the middle of the night and without turning on the light chooses a jar at random and then chooses a cookie at random. If the cookie is chocolate-chip, what is the probability that he got the cookie from the blue jar?

Exercise 10.6 Ten percent of the pens made by Apex are defective. Only 5% of the pens made by its competitor, B-ink, are defective. Since Apex pens are cheaper than B-ink pens, an office order 70% of its stock from Apex and 30% from B-ink. A pen is chosen at random and found to be defective. What is the probability that it was produced by Apex?

Credit

This lecture note is from Assoc. Prof. Dr. Nirattaya Khamsemanan's GTS111 partial note.